12.6 Cylinders and Quadric Surfaces (page 834)

Cylinders, page 834

Definition 1 (page 827)**.** A *cylinder* (柱面) is a surface that consists of all lines (called *rulings*, 母線) that are parallel to a given line and pass through a given plane curve.

☐ 以上述定義, 柱面是更一般的概念, 不限定是「圓柱面」。

Example 2 (page 834)**.** The following surfaces are cylinders:

(a) Circular cylinder: $x^2 + y^2 = 1$. The rulings are parallel to the z-axis.

(b) parabolic cylinder: $z = x^2$. The rulings are parallel to the y-axis.

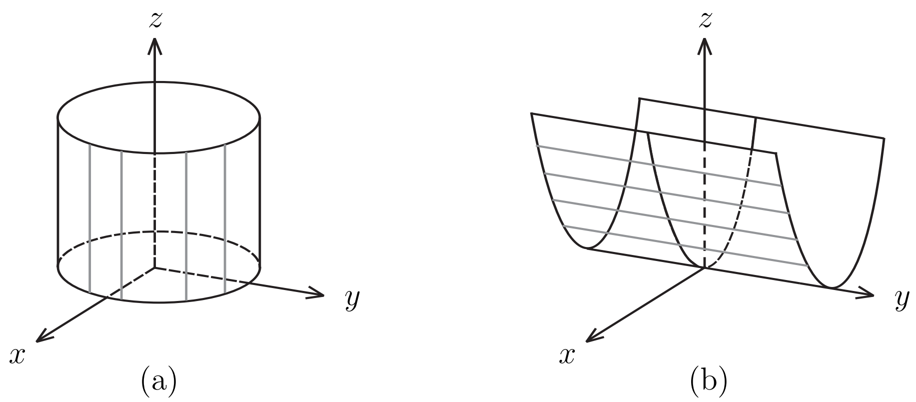

Figure 1: (a) Circular cylinder. (b) Parabolic cylinder.

Quadric Surfaces, page 835

Definition 3 (page 835)**.** A *quadric surfaces* (二次曲面) is the graph of a second-degree equation in three variables x, y, an z. The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, \ldots, J$ are constants.

(a) If $A = B = C = D = E = F = 0$ and one of G, H, I is nonzero, then the surface is a plane.

(b) If one of A, B, C, D, E, F is nonzero, by translation and rotation, it can be brought into one of the two standard forms

$$Ax^2 + By^2 + Cz^2 + J = 0 \qquad \text{or} \qquad Ax^2 + By^2 + Iz = 0.$$

☐ 在代數上,「旋轉」是將交叉項 D, E, F 消除, 而平移是用「配方法」達成。

Six types of quadric surfaces in standard form, page 837

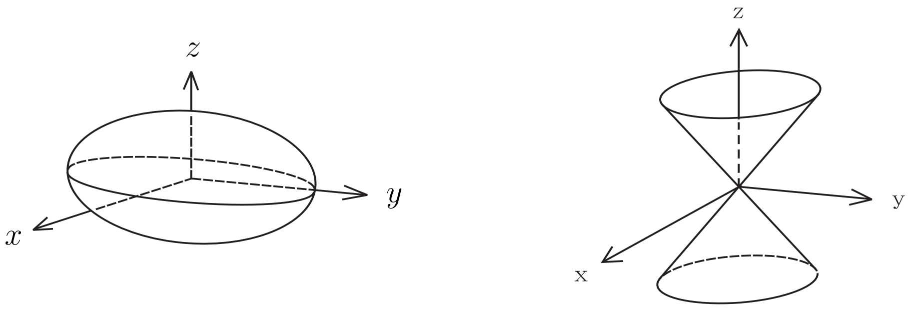

Figure 2: Ellipsoid (橢球) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and cone (錐) $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

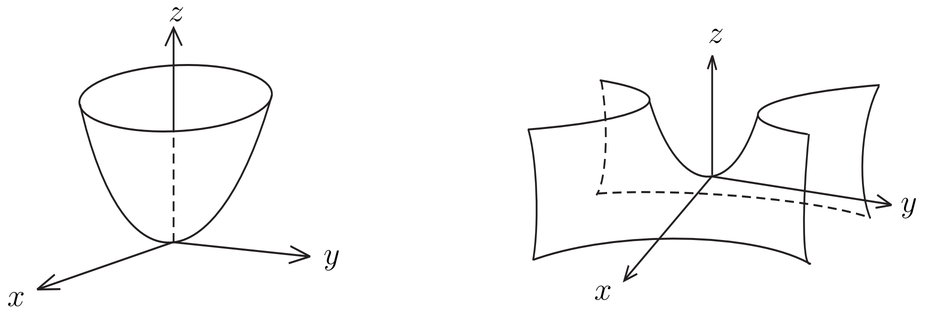

Figure 3: Elliptic paraboloid (橢圓拋物面) $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ and hyperbolic paraboloid (雙曲拋物面) $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}, c < 0$.

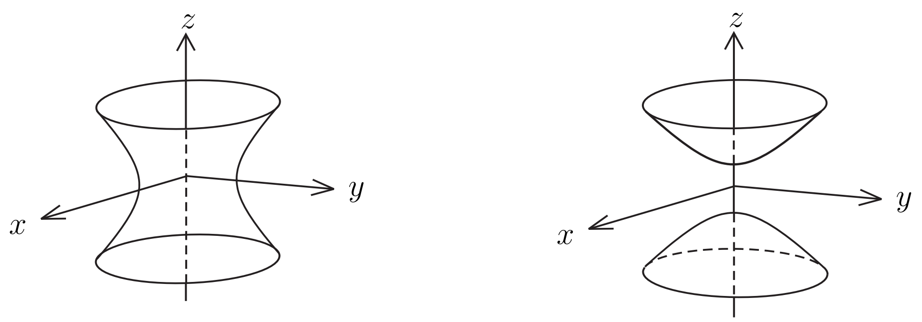

Figure 4: Hyperboloid of one sheet (單葉雙曲面) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ and hyperboloid of two sheets (雙葉雙曲面) $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Exercise (page 840)**.** Classify the following surfaces.

(a) $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$.

(b) $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$.

(c) $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.

(d) $z = x^2 - y^2$. (e) $y^2 + z^2 = 1 + x^2$. (f) $-4x^2 + y^2 - 4z^2 = 4$.