9.4 Models for Population Growth (page 610)

Other Models for Population Growth, page 617

The Law of Natural Growth and the logistic differential equation are not the only equations that have been proposed to model population growth.

Example 1 (page 620)**.** Another model for a growth function for a limited population is given by the *Gompertz function*, which is a solution of the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = c \ln\left(\frac{M}{P}\right) P,$$

where c is a constant and M is the carrying capacity.

Example 2 (page 620)**.** In a *seasonal-growth model*, a periodic function of time is introduced to account for seasonal variations in the rate of growth. Such variations could, for example, be caused by seasonal changes in the availability of food. For example, we can consider the seasonal-growth model

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\cos(rt - \phi), \quad P(0) = P_0,$$

where k, r, and ϕ are positive constants. We can alter the differential equation as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\cos^2(rt - \phi), \quad P(0) = P_0,$$

Example 3 (page 617, 619)**.** The differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{M}\right) - c$$

has been used to model populations that are subject to harvesting of one sort or another. (Think of a population of fish being caught at a constant rate.)

Example 4 (page 617, 620)**.** For some species there is a minimum population level m below which the species tends to become extinct. Such populations have been modeled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right),$$

where the extra factor $1 - \frac{m}{P}$, take into account the consequences of a sparse population.