7.3 Trigonometric Substitution (page 486)

Trigonometric identities are also useful to make substitutions for some radical functions.

Table of Trigonometric Substitutions.

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,\ 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Example 1 (page 487)**.** Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Solution.

Example 2 (page 490)**.** Find $\displaystyle\int \frac{x}{\sqrt{3 - 2x - x^2}}\,\mathrm{d}x$.

Solution.

Exercise (page 486)**.** Evaluate $\displaystyle\int \frac{\sqrt{9 - x^2}}{x^2}\,\mathrm{d}x$.

Example 3 (page 488). Find $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,\mathrm{d}x$.

Solution.

Exercise. Evaluate the integral $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{1+x^2}}\,\mathrm{d}x$.

Example 4 (page 489). Find $\displaystyle\int \frac{1}{\sqrt{x^2-a^2}}\,\mathrm{d}x$, where $a > 0$.

Solution.

Exercise. Find the integral $\displaystyle\int \frac{2}{x^3\sqrt{x^2-1}}\,\mathrm{d}x, \quad x > 1$.

Exercise. Evaluate the integral $\displaystyle\int \frac{x}{\sqrt{x^2+2x+2}}\,\mathrm{d}x$.

Example 5 (page 490). Find $\displaystyle\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{\frac{3}{2}}}\,\mathrm{d}x$.

Solution.