6.5 Average Value of a Function (page 461)

Definition 1 (page 461)**.** We define the *average value of* f (平均値) on the interval $[a, b]$ as

$$f_{\text{ave}} = \lim_{n\to\infty} \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\Delta x = \frac{1}{b-a} \int_a^b f(x)\,\mathrm{d}x.$$

Example 2. Find the average of $f(x) = \sin x$ on $[0, \pi]$.

Solution.

The Mean Value Theorem for Integrals (pgae 462)**.** *If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that*

$$f(c) = f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x)\,\mathrm{d}x. \qquad \left(or \int_a^b f(x)\,\mathrm{d}x = f(c)(b-a). \right)$$

Proof. Consider $F(x) = \int_a^x f(t)\,\mathrm{d}t$. Since $f(x)$ is continuous on $[a, b]$, $F(x)$ is continuous on $[a, b]$ and differentiable on (a, b). By the Mean Value Theorem, there exists $c \in (a, b)$ such that _____. By the Fundamental Theorem, we have _____. Hence

$$f(c) =$$

□

Example 3. Suppose that $f(x)$ is an increasing and continuous function on $[a, b]$. Find the line $y = L$ such that $\int_a^b |f(x) - L|\,\mathrm{d}x$ is minimum.

Solution.

Example 4. Suppose that $f(x)$ is a continuous function $f(x)$ on $[a, b]$. Find the line $y = L$ such that $\int_a^b (f(x) - L)^2 \, \mathrm{d}x$ is minimum.

Solution.

☐ 中位數 (median) 與平均數 (average) 的意義。