6.3 Volumes by Cylindrical Shells (page 449)

Definition 1 (page 437)**.** The volume of the solid obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b is

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} 2\pi \bar{x}_i f(\bar{x}_i) \Delta x = \int_{a}^{b} 2\pi x f(x) \, dx, \quad \text{where} \ \ 0 \le a < b.$$

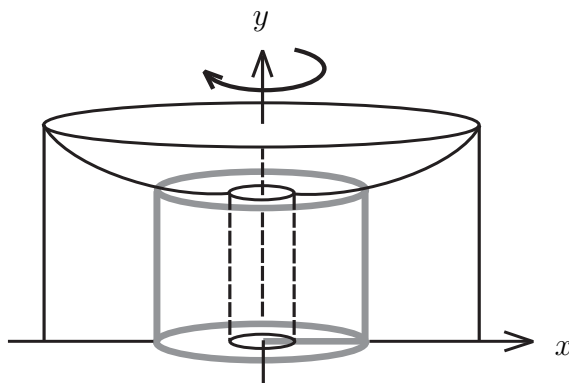This method is called *cylindrical shells method* (柱殼法).

Figure 1: The volume formula by cylindrical shells (rotate about y-axis).

Example 2 (page 453)**.** Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = \sin(x^2)$ and $y = 0$ for $0 \le x \le \sqrt{\pi}$.

Solution.

Example 3. Find the volume of the solid obtained by rotating about $x = -1$ the region bounded by $y = 6x^2, x = 1$, and $y = 0$.

Solution.

☐ 計算旋轉體體積問題, 除了區域要確定以外, 對哪一個軸旋轉也很重要。

☐ 目前無法解決太多用柱殼法所列出的體積問題, 這是因爲尚有一些積分技巧未學。

The volume formula of solid of revolution

(a) Region under $f(x) > 0, x \in [a, b]$; rotate about y-axis.

(b) Region under $f(x) > 0, x \in [a, b]$; rotate about $x = c, c < a$.