6.2 Volumes (page 438)

Definition 1 (page 439)**.** Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the *volume* (體積) of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b A(x)\,\mathrm{d}x.$$

Definition 2 (page 443)**.** The solids are obtained by revolving a region about a line is called *solids of revolution* (實心旋轉體).

In general, we calculate the volume of a solid of revolution by the formula

$$V = \int_a^b A(x)\,\mathrm{d}x \quad \text{or} \quad V = \int_c^d A(y)\,\mathrm{d}y,$$

where

- If the cross-section is a disk, then $A = \pi(\text{radius})^2$.
- If the cross-section is a washer, then $A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$.

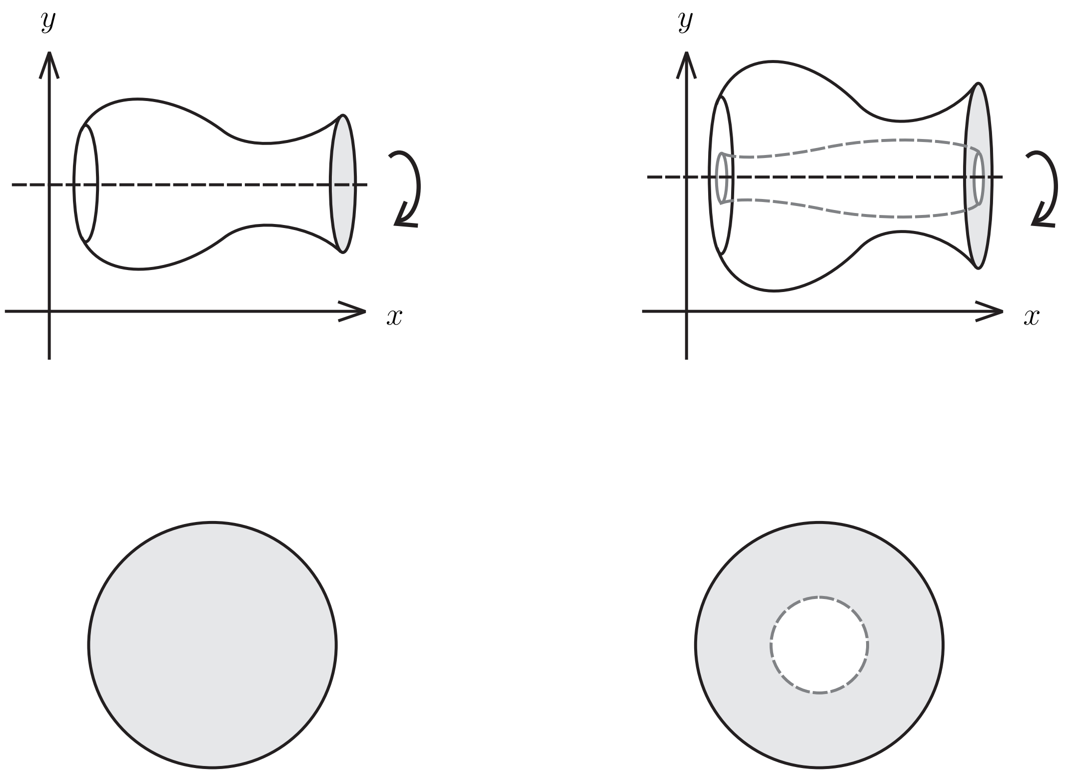

Figure 1: The volume formula of solids of revolution.

Example 3 (page 439)**.** Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

Solution.

Example 4 (page 448)**.** Compute the volume of the solid torus.

Solution.

Example 5 (page 442)**.** Consider the region $\mathcal{R}$ enclosed by the curves $y = x$ and $y = x^2$.

(a) Find the volume of the solid obtained by rotating the region about the line $y = 2$.

(b) Find the volume of the solid obtained by rotating the region about the line $x = -1$.

Solution.

We now find the volumes of two solids that are *not* solids of revolution.

Example 6 (page 445)**.** Find the volume of a pyramid whose base is a square with side L and whose height is h.

Solution.

Example 7 (page 446)**.** A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of $30°$ along a diameter of the cylinder. Find the volume of the wedge.

Solution.

Example 8 (page 449)**.** Find the volume common to two circular cylinders, each with radius r, if the axis of the cylinder intersect at right angles.

Solution.

The volume formula of solid of revolution

(a) Region under $f(x) > 0$; rotate about x-axis.

(b) Region between $f(x)$ and $g(x)$, $f(x) > g(x) > 0$; rotate about x-axis.

(c) Region under $f(x) > 0$; rotate about the line $y = c$.

(d) Region between $f(x)$ and $g(x)$, $f(x) > g(x) > c$; rotate about the line $y = c$.