Chapter 6 Applications of Integration

6.1 Areas Between Curves (page 422)

Theorem 1 (page 422)**.** *The area A of the region bounded by the curves $y = f(x), y = g(x)$, and the lines $x = a, x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$, is*

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i^*) - g(x_i^*))\Delta x = \int_a^b (f(x) - g(x))\,\mathrm{d}x.$$

Proof. This is because

$$\begin{aligned} A &= (\text{area under } y = f(x)) - (\text{area under } y = g(x)) \\ &= \int_a^b f(x)\,\mathrm{d}x - \int_a^b g(x)\,\mathrm{d}x = \int_a^b (f(x) - g(x))\,\mathrm{d}x. \end{aligned}$$

□

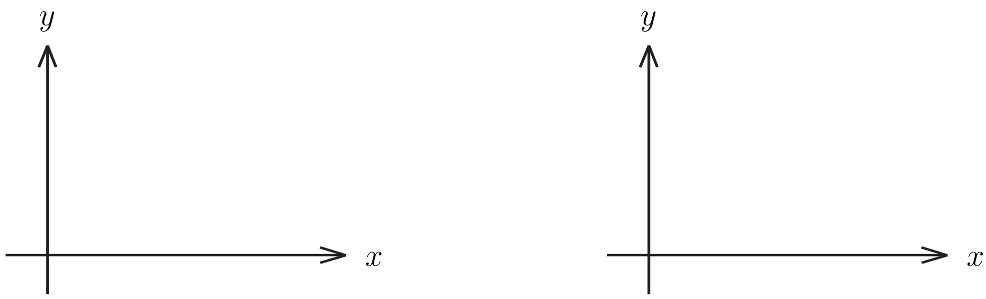
Theorem 2 (page 425)**.** *The area between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is*

$$A = \int_a^b |f(x) - g(x)|\,\mathrm{d}x.$$

Proof. This is because

$$|f(x) - g(x)| = \begin{cases} f(x) - g(x) & \text{if } f(x) \geq g(x) \\ g(x) - f(x) & \text{if } g(x) \geq f(x) \end{cases}.$$

□



Area =

Area =

Figure 1: The area formula.

Example 3 (page 427)**.** Sketch the region enclosed by $y = \tan x, y = 2\sin x, -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$ and find its area.

Solution.

Example 4 (page 426)**.** Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Solution.