5.4 Indefinite Integrals and the Net Change Theorem (page 402)

Table of Indefinite Integrals (page 403).

$$\int cf(x)\,\mathrm{d}x = c\int f(x)\,\mathrm{d}x \qquad \int (f(x)+g(x))\,\mathrm{d}x = \int f(x)\,\mathrm{d}x + \int g(x)\,\mathrm{d}x$$

$$\int k\,\mathrm{d}x = kx + C$$

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C.$$

$$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C \qquad \int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C.$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C \qquad \int \cos x\,\mathrm{d}x = \sin x + C.$$

$$\int \sec^2 x\,\mathrm{d}x = \tan x + C \qquad \int \csc^2 x\,\mathrm{d}x = -\cot x + C.$$

$$\int \sec x \tan x\,\mathrm{d}x = \sec x + C \qquad \int \csc x \cot x\,\mathrm{d}x = -\csc x + C.$$

$$\int \frac{1}{x^2+1}\,\mathrm{d}x = \tan^{-1} x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \sin^{-1} x + C.$$

$$\int \sinh x\,\mathrm{d}x = \cosh x + C \qquad \int \cosh x\,\mathrm{d}x = \sinh x + C$$

We adopt the convention that when a formula for a general indefinite integral is given, it is valid only on an interval. For example, the general antiderivative of the function $f(x) = \frac{1}{x}, x \neq 0$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if } x > 0 \\ \ln|x| + C_2 & \text{if } x < 0 \end{cases}.$$

☐ A definite integral $\int_a^b f(x)\,\mathrm{d}x$ is a number; an indefinite integral $\int f(x)\,\mathrm{d}x$ is a function (family of functions).

Applications

Recall that the Fundamental Theorem of Calculus, part 2:

The Fundamental Theorem of Calculus, Part 2 (page 396)**.** *If f is continuous on $[a,b]$, then*

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a),$$

where $F(x)$ is any antiderivative of $f(x)$, that is, a function such that $F'(x) = f(x)$.

We put $f(x) = F'(x)$ into the Theorem and get

Net Change Theorem (page 406)**.** *The integral of a rate of change is the net change:*

$$\int_a^b F'(x)\,\mathrm{d}x = F(b) - F(a).$$

This principle can be applied to all of the rates of change in the natural and social sciences. For example,

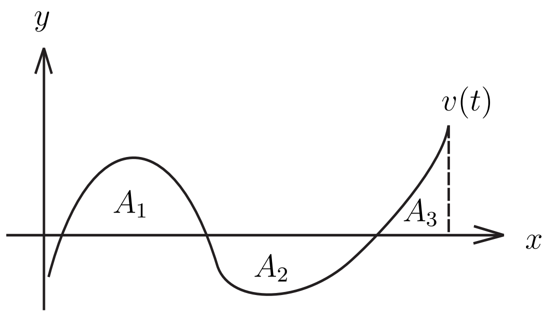
- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$, so

  $$\int_{t_1}^{t_2} v(t)\,\mathrm{d}t = s(t_2) - s(t_1)$$

  is the net change of position, or *displacement*, of the particle during the time period from t_1 to t_2.

  If we want to calculate the distance the object travels during the time interval, we have to consider the intervals when $v(t) \geq 0$ and also the intervals when $v(t) \leq 0$. In both cases the distance is computed by integrating $|v(t)|$, the speed. Therefore,

  $$\int_{t_1}^{t_2} |v(t)|\,\mathrm{d}t = \text{total distance traveled}.$$



Displacement =

Distance =

Figure 1: Displacement and distance.

- The acceleration of the object is $a(t) = v'(t)$, so

  $$\int_{t_1}^{t_2} a(t)\,\mathrm{d}t = v(t_2) - v(t_1)$$

  is the change in velocity from time t_1 to time t_2.