Chapter 5 Integrals

5.1 Areas and Distances (page 360)

The Area Problem, page 360

Example 1. Use rectangles to estimate the area under the parabola $y = x^2$ from 0 to 1.

Solution.

Definition 2 (page 365)**.** The *area* (面積) A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} (f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x).$$

It can also be shown that we get the same value if we use left endpoints:

$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} (f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x).$$

In fact, instead of using left endpoints or right endpoints, we could take the height of the i-th rectangle to be the value of f at *any* number x_i^* in the i-th subinterval $[x_{i-1}, x_i]$. We call numbers $x_1^*, x_2^*, \ldots, x_n^*$ the *sample points* (樣本點).

In general, we form *lower sums* (下和) (and *upper sums*, 上和) by choosing the sample points x_i^* so that $f(x_i^*)$ is the minimum (and maximum) value of f on the i-th subinterval.

The Distance Problem, page 367

We can find the distance traveled by an object during a certain time period if the velocity of the object is known at all times.