4.9 Antiderivative (page 350)

Definition 1 (page 350)**.** A function F is called an *antiderivative* (反導函數) of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem 2 (page 351)**.** *If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is*

$$F(x) + C,$$

where C is an arbitrary constant.

Proof. If F and G are any two antiderivative of f, then $F'(x) = f(x) = G'(x)$. Form the corollary of the Mean Value Theorem (Section 4.2 Corollary 8), we know $G(x) - F(x) = C$, where C is a constant. So $G(x) = F(x) + C$. □

This is a table of antidifferentiation formulas. We use the notation $F'(x) = f(x)$ and $G'(x) = g(x)$.

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sec^2 x$	$\tan x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec x \tan x$	$\sec x$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$\frac{1}{x}$	$\ln \lvert x \rvert$	$\frac{1}{1+x^2}$	$\tan^{-1} x$
e^x	e^x	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$
$\sin x$	$-\cos x$		

Example 3. Find the most general antiderivative of the function. (Let $F(x)$ is the antiderivative of the function $f(x)$.)

(1) $f(x) = e^2$ $F(x) =$

(2) $f(x) = x(2-x)^2$

$F(x) =$

(3) $f(x) = x^{\pi} - x^{3.14}$ $F(x) =$

(4) $f(x) = \frac{2+x^2}{1+x^2}$

$F(x) =$

Example 4. Find $f(x)$.

(1) $f'(x) = 2\cos x + \sec^2 x,\ -\frac{\pi}{2} < x < \frac{\pi}{2},\ f(\frac{\pi}{3}) = 4.$

(2) $f''(x) = 2\mathrm{e}^x + 3\sin x, f(0) = 0, f(\pi) = 0.$

Solution.

☐ 以上方程式稱爲「帶有初始條件的微分方程式」(ordinary differential equation)。