Chapter 4 Applications of Differentiation

4.1 Maximum and Minimum Values (page 276)

Definition 1 (page 276). Let c be a number in the domain D of a function f. Then $f(c)$ is the

(1) *absolute maximum value* (絕對極大值) of f on D if $f(c) \geq f(x)$ for all x in D.

(2) *absolute minimum value* (絕對極小值) of f on D if $f(c) \leq f(x)$ for all x in D.

☐ Absolute maximum (or minimum) 有時候也稱爲 *global maximum* (or *minimum*).

☐ 所有的絕對極大值、絕對極小值統稱爲函數 f 的極值 (*extreme values*).

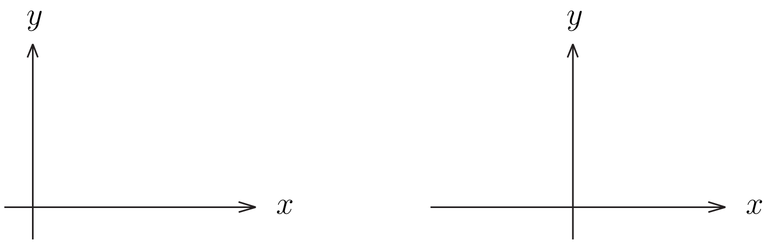

Figure 1: Absolute maximum value and absolute minimum value of f.

☐ 判斷絕對極值時, 必須定義域內「所有」(for all) 的點都要做比較。

Definition 2 (page 276). The number $f(c)$ is a

(1) *local maximum value* (局部極大值) of f on D if $f(c) \geq f(x)$ when x is near c.

(2) *local minimum value* (局部極小值) of f on D if $f(c) \leq f(x)$ when x is near c.

We say that something is true *near* c, we mean that it is true on "some *open* interval containing c."

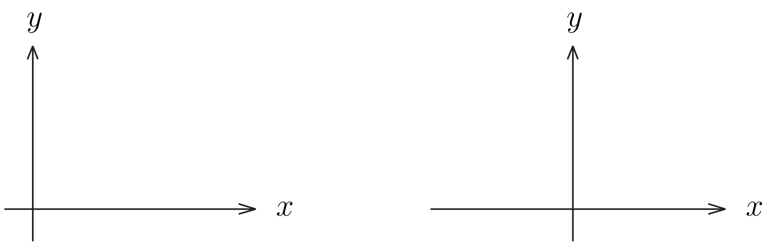

Figure 2: Local maximum value and local minimum value of f.

☐ 局部極值的定義中,「附近」(near) 這個詞很重要。

Example 3. State the absolute (and local) maximum (and minimum) values of the function $y = f(x)$.

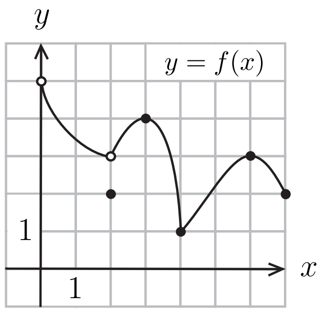

Figure 3: Find absolute (and local) maximum (and minimum) values of the function.

Solution.

(a) Absolute maximum:

(b) Local maximum:

(c) Absolute minimum:

(d) Local minimum:

Theorem 4 (The Extreme Value Theorem, 極值定理, page 278)**.** *If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute value $f(d)$ at some numbers c and d in $[a, b]$.*

☐ 極值定理的條件是「閉區間」上的「連續函數」。

☐ 極值定理的結論只告知「存在性」。

Example 5. Give examples that if f is not continuous, or f is continuous on (a, b), the Extreme Value Theorem does not hold. Plot a continuous function that it attains maximum values and minimum values at more than one numbers.

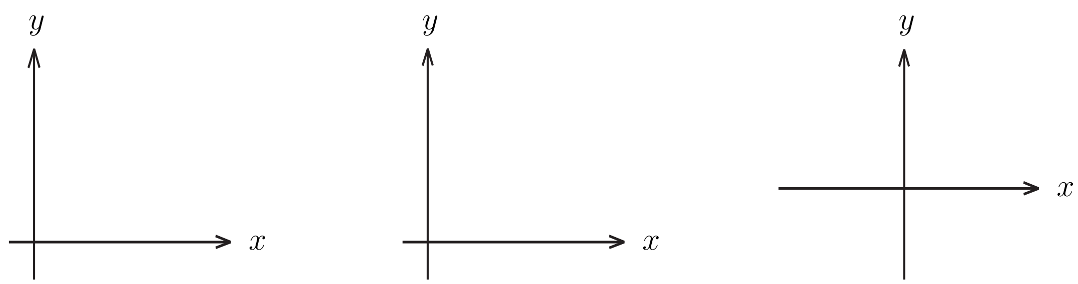

Figure 4: Study the Extreme Value Theorem.

Theorem 6 (Fermat's Theorem 費馬定理, page 279)**.** *If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.*

☐ 條件 "$f'(c)$ exists" 很重要。反例: ____________.

☐ 一般而言, 費馬定理的反敘述不對, 例如: ____________.

Proof. Here we prove the local maximum case. Since $f(c) \geq f(x)$ if x is sufficiently close to c, this implies that if h is sufficiently close to 0, with h being positive or negative, then $f(c) \geq f(c+h)$, or equivalently, $f(c+h) - f(c) \leq 0$.
<u>If $h > 0$</u>, we have $\frac{f(c+h)-f(c)}{h} \leq 0$. Since $f'(c)$ exists, we get

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} =$$

<u>If $h < 0$</u>, we have $\frac{f(c+h)-f(c)}{h} \geq 0$. Since $f'(c)$ exists, we get

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} =$$

Hence $f'(c) = 0$. ☐

Definition 7 (page 280)**.** A *critical number* (臨界點) of a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.

☐ 臨界點 (critical numbers) 是在函數 f 在「定義域」內。

Example 8 (page 280)**.** Find the critical numbers of $f(x) = x^{\frac{3}{5}}(4-x) = 4x^{\frac{3}{5}} - x^{\frac{8}{5}}$.

Solution. We compute $f'(x) =$
Therefore the critical numbers are ________.

Theorem 9 (page 280)**.** *If f has a local maximum or minimum at c, then c is a critical number of f.*

☐ 此定理等價於"若函數 f 不存在臨界點, 則 f 沒有局部極大值, 也沒有局部極小值。"

The Closed Interval Method (page 280)**.** To find the *absolute* maximum and minimum values of a piecewise continuous function on a closed interval $[a, b]$:

(1) Find the values of f at the critical numbers of f in (a, b).

(2) Find the values of f at the endpoints of the interval, that is, $f(a)$ and $f(b)$.

(3) The largest and smallest of the values from (1) and (2) are absolute maximum value and absolute minimum value, respectively.

☐ 求絕對極值的方法: 找出所有臨界點與端點; 比較那些點的函數值之大小。