3.9 Related Rates (page 245)

Idea: Compute the rate of increase of one quantity in terms of the rate of change of another quantity (which may be more easily measured).

Procedure:

(1) Draw a picture or a diagram if possible.

(2) Introduce notation. Assign symbols to all quantities that are functions of time.

(3) Find an equation that relates the two quantities and then use the Chain Rule to differentiate both side with respect to time.

(4) Substitute the given information into the equation and get the unknown rate.

Example 1 (page 245). Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\,\mathrm{cm^3/s}$. How fast is the radius of the balloon increasing when the diameter is $50\,\mathrm{cm}$?

Solution.

Example 2 (page 246). A ladder $5\,\mathrm{m}$ long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $1\,\mathrm{m/s}$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is $3\,\mathrm{m}$ from the wall?

Solution.

□ $\frac{\mathrm{d}y}{\mathrm{d}t}$ 的 "負號" 代表梯頂離地以 $\frac{3}{4}\,\mathrm{m/s}$ 之變化率 "減少"。

Example 3 (page 246). A water tank has the shape of an inverted circular cone with base radius $2\,$m and height $4\,$m. If water is being pumped into the tank at a rate of $2\,\text{m}^3/\text{min}$, find the rate at which level is rising the water is $3\,$m deep.

Solution.

The water of level is rising at a rate of _____ .

Example 4 (page 247). Car A is traveling west at $90\,$km/h and car B is traveling north at $100\,$km/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is $60\,$m and car B is $80\,$m from the intersection?

Solution.

The cars are approaching each other at a rate of _____ .

Example 5 (page 248). A man walks along a straight path at a speed of $1.5\,$m/s. A searchlight is located on the ground $6\,$m from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is $8\,$m from the point on the path closest to the searchlight?

Solution.

The searchlight is rotating at a rate of _____ .