

## 1.3 New Functions from Old Functions, page 36

Given a function  $f(x)$ , we will discuss the new function

$$g(x) = af(bx + c) + d$$

for constants  $a, b, c, d$  and the effects of these constants.

- The effect of  $a$  is vertical dilation (上下脹縮, 對  $x$ -軸).
- The effect of  $b$  is horizontal dilation (左右脹縮, 對  $y$ -軸).
- The effect of  $c$  is horizontal shift (左右平移).
- The effect of  $d$  is vertical shift (上下平移).

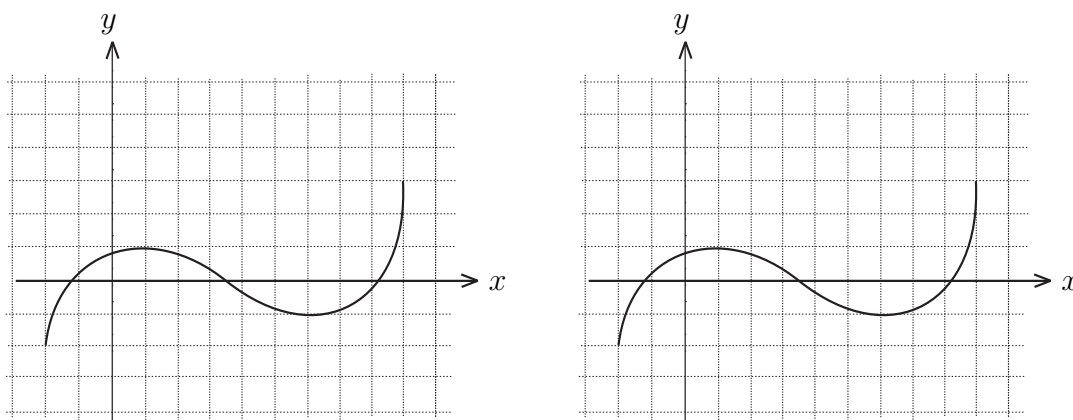


Figure 1: The effect of  $d = 2$  and  $a = 2$ .

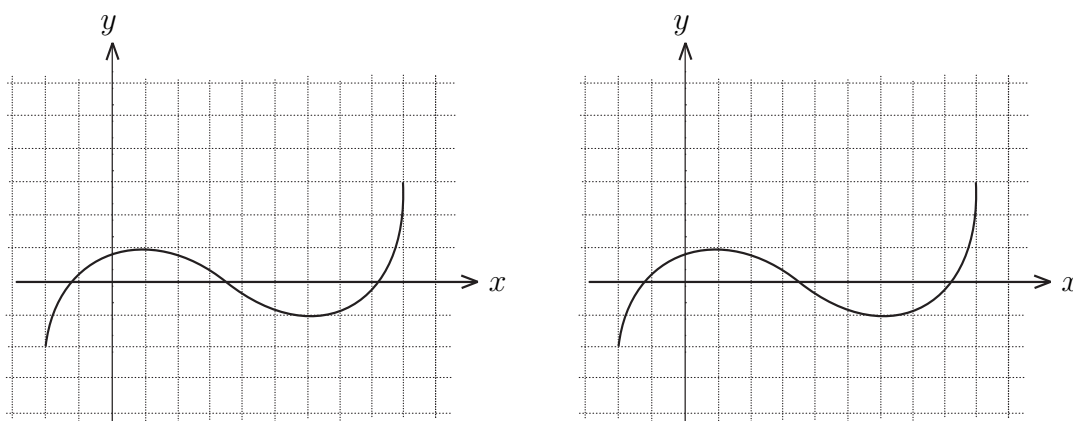


Figure 2: The effect of  $c = -1$  and  $b = 2$ .

- $0 < b < 1$ : 橫向膨脹。  $b > 1$ : 橫向壓縮。  $c < 0$ : 向右平移。  $c > 0$ : 向左平移。
- 在函數裡面的係數效應要注意 (有“相反”的意味)。
- 從  $f(x)$  要畫出  $af(bx + c) + d$ , 處理順序為  $c \rightarrow b \rightarrow a \rightarrow d$  或  $a \rightarrow d \rightarrow c \rightarrow b$ 。

**Example 1.** Starting from  $f(x) = x^2$ , plot the function  $f_4(x) = \frac{1}{3}(2x + 5)^2 - 1$ .

**Solution.**

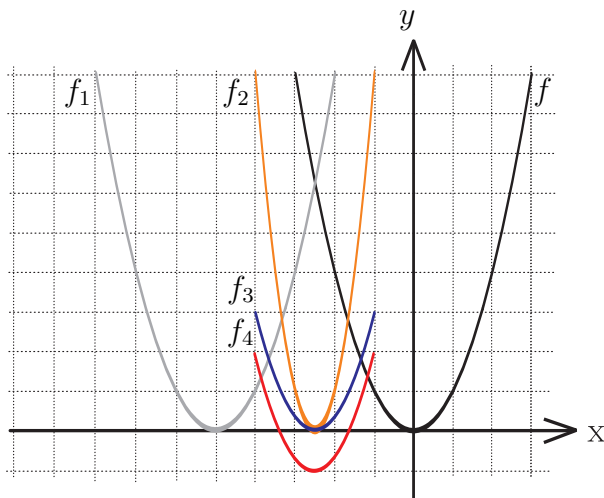


Figure 3:  $f(x) = x^2$ ,  $f_1(x) = (x + 5)^2$ ,  $f_2(x) = (2x + 5)^2$ ,  $f_3(x) = \frac{1}{3}(2x + 5)^2$ , and  $f_4(x) = \frac{1}{3}(2x + 5)^2 - 1$ .

**Definition 2** (page 41). Given two functions  $f$  and  $g$ , the *composite function*  $f \circ g$  (also called the *composition* of  $f$  and  $g$ ) (合成函數) is defined by

$$(f \circ g)(x) = f(g(x)).$$

□ 記號  $f \circ g$  是先作用  $g$ , 再作用  $f$ 。

□ 一般而言,  $f \circ g \neq g \circ f$ 。

**Example 3** (Composition of a function and the absolute value function). Discuss the relations between  $f(x)$ ,  $|f(x)|$ , and  $f(|x|)$ .

**Solution.**

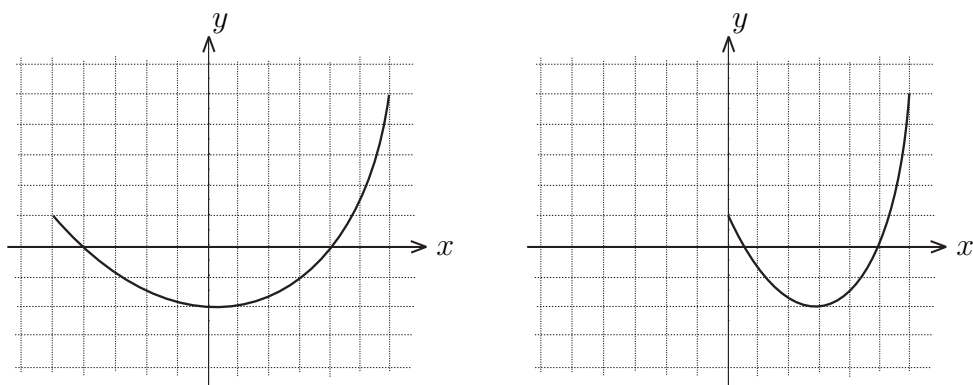


Figure 4: Left: Given  $f(x)$ , plot  $|f(x)|$ . Right: Given  $f(x)$ , plot  $f(|x|)$ .

**Example 4.** Given  $f(x)$ , compare the process  $c \rightarrow b \rightarrow a \rightarrow d$  with  $b \rightarrow c \rightarrow a \rightarrow d$ .

**Solution.** Given a function  $f(x)$ , when we consider steps  $c \rightarrow b \rightarrow a \rightarrow d$ , it would be

$$f_1(x) =$$

$$f_2(x) =$$

$$f_3(x) =$$

$$f_4(x) =$$

When we consider steps  $b \rightarrow c \rightarrow a \rightarrow d$ , it would be

$$\tilde{f}_1(x) =$$

$$\tilde{f}_2(x) =$$

$$\tilde{f}_3(x) =$$

$$\tilde{f}_4(x) =$$

- 先後順序不同 ( $b \rightarrow c$  與  $c \rightarrow b$ , 還有  $a \rightarrow d$  與  $d \rightarrow a$ ) 會導致不同的效果。
- 新函數  $af(bx + c) + d$  是由原函數  $f(x)$  與一系列函數作合成而得。