# Chapter 1 Functions and Models

## 1.1 Four Ways to Represent a Function, page 10

**Definition 1** (page 10)**.** A *function* (函數) $f$ is a rule that assigns to each element $x$ in a set $D$ exactly one element, called $f(x)$ in a set $E$.

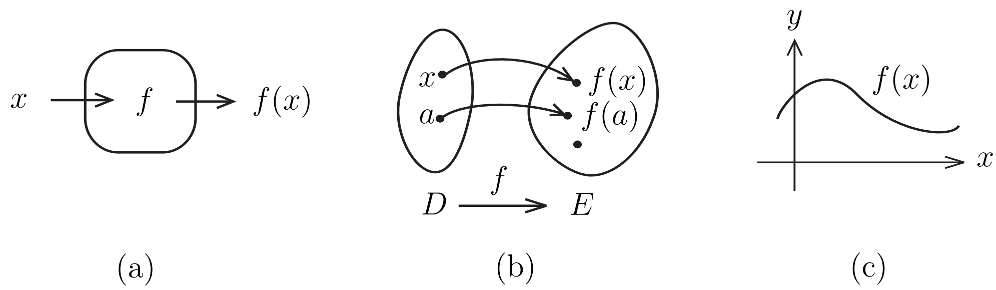


Figure 1: (a) machine diagram; (b) arrow diagram; (c) graph (圖形) of a function.

We usually consider functions for which the sets $D$ and $E$ are sets of real numbers $\mathbb{R}$.

☐ domain (定義域), codomain (對應域), range (值域).

☐ value of $f$ at $x$ (or "$f$ of $x$").

☐ independent variable, dependent variable.

There are four possible ways to represent a function:

(1) verbally (by a description in words). 平常的溝通與交流

(2) numerically (by a table of values). 透過數據之觀察可發現一些現象

(3) visually (by a graph). 視覺引導通常會帶來深刻印象, 但有時圖形無法如實呈現

(4) algebraically (by an explicit formula). 數學上的嚴謹性充足, 但有時候不直覺

**Example 2.** Find the domain of the following function:

(1) $f(x) = \frac{x^2}{1+x}$. $D = \{x \in \mathbb{R} \mid \qquad \}$.

(2) $f(x) = (x-2)\sqrt{\frac{1+x}{1-x}}$. $D = \{x \in \mathbb{R} \mid \qquad \}$.

(3) $f(x) = \log(x+2) + \log(x-2)$. $D = \{x \in \mathbb{R} \mid \qquad \}$.

(4) $f(x) = \tan x$. $D = \{x \in \mathbb{R} \mid \qquad \}$.

**Vertical Line Test** (page 15)**.** *A curve in the xy-plane is the graph of a function of $x$ if and only if no vertical line intersects the curve more than once.*

☐ 判斷一條曲線是否爲函數的圖形, 幾何上使用「垂直線法」。

**Example 3.** Give examples that one curve is the graph of a function and one curve is not the graph of a function.


Figure 2: Left curve is a graph of a function; Right curve is not a graph of a function.

**Example 4** (page 16)**.** The *absolute value* (絕對值) of a number $a$, denoted by $|a|$, is the distance from $a$ to 0 on the real number line. The graph of the absolute value function is

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

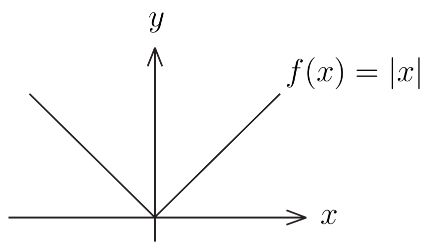


Figure 3: The graph of the absolute value function.

**Example 5.** Sketch the graph of the *Heaviside function* $H(x)$, which is defined by

$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

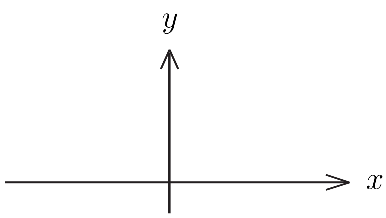


Figure 4: The Heaviside function.

**Example 6.** The graph of the *sign function* $\operatorname{sgn}(x)$ (符號函數), which is defined by

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0. \end{cases}$$

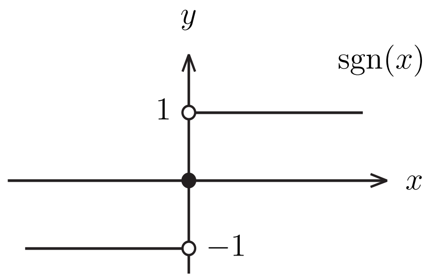


Figure 5: The sign function $\operatorname{sgn}(x)$.

☐ 稱之爲符號函數的原因是: ______________________

**Definition 7** (Odd function and even function, page 17–18)**.**

(a) If a function $f$ satisfies $f(-x) = -f(x)$ for every number $x$ in the domain, then $f$ is called an *odd function* (奇函數).

(b) If a function $f$ satisfies $f(-x) = f(x)$ for every number $x$ in the domain, then $f$ is called an *even function* (偶函數).

☐ 奇函數的例子: ___________________________________

☐ 偶函數的例子: ___________________________________

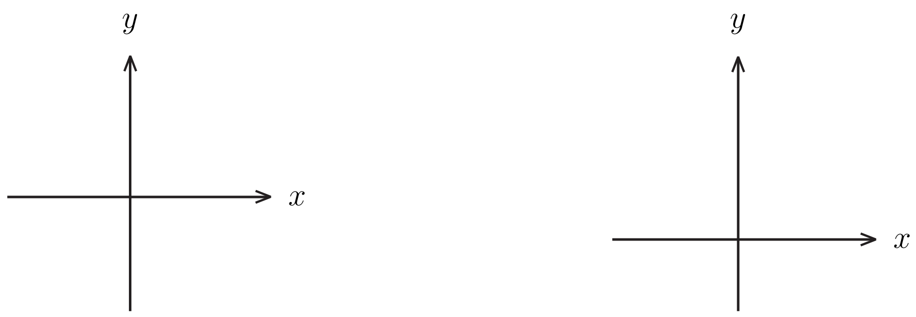


Figure 6: Left: odd function; Right: even function.

☐ 所有奇函數圖形必對稱於 _______

☐ 所有偶函數圖形必對稱於 _______

**Example 8.** Any function defined on $\mathbb{R}$ can be (uniquely) decomposed as the sum of an odd function and an even function.

*Proof.* Define two functions

$$g(x) = \frac{f(x) - f(-x)}{2} \quad \text{and} \quad h(x) = \frac{f(x) + f(-x)}{2}.$$

We will show that

- $g(x)$ is an odd function:

- $h(x)$ is an even function:

- $f(x) = g(x) + h(x)$:

□

**Definition 9** (Increasing and decreasing functions, page 19)**.**

(a) A function $f(x)$ is called *increasing* (遞增) on an interval $I$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in $I$.

(b) A function $f(x)$ is called *decreasing* (遞減) on an interval $I$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in $I$.


Figure 7: Increasing function and decreasing function.

☐ 教科書與微積分課用 increasing 及 decreasing 等詞彙時, 函數值比較都是「不等號」。

☐ 有些書或文獻會用 “strictly” 或 “monotone” increasing (decreasing) 強調不等號。