

12.6 Cylinders and Quadric Surfaces (page 834)

Cylinders, page 834

Definition 1 (page 827). A *cylinder* (柱面) is a surface that consists of all lines (called *rulings*, 母線) that are parallel to a given line and pass through a given plane curve.



q837hF7jM74

□ 以上述定義，柱面是更一般的概念，不限定是「圓柱面」。

Example 2 (page 834). The following surfaces are cylinders:

- (a) Circular cylinder (圓柱面): $x^2 + y^2 = 1$. The rulings are parallel to the z -axis.
- (b) Parabolic cylinder (拋物柱面): $z = x^2$. The rulings are parallel to the y -axis.

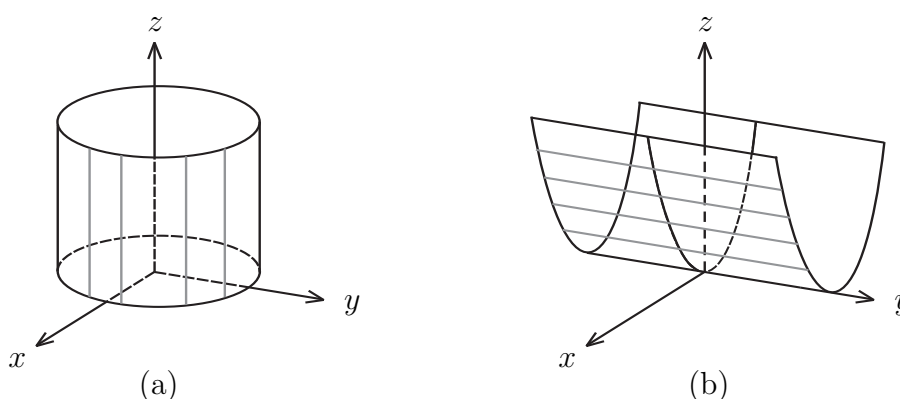


Figure 1: (a) Circular cylinder. (b) Parabolic cylinder.

Quadric Surfaces, page 835

Definition 3 (page 835). A *quadric surfaces* (二次曲面) is the graph of a second-degree equation in three variables x, y , and z . The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where A, B, C, \dots, J are constants.

- (a) If $A = B = C = D = E = F = 0$ and one of G, H, I is nonzero, then the surface is a plane.
- (b) If one of A, B, C, D, E, F is nonzero, by translation and rotation, it can be brought into one of the two standard forms

$$Ax^2 + By^2 + Cz^2 + J = 0 \quad \text{or} \quad Ax^2 + By^2 + Iz = 0.$$

通常我們口語在講柱面或是看到英文 cylinder 這個詞時，都會很直覺地聯想成圓柱。實際上柱面這個數學定義很一般，只要是母線沿著一條平面曲線平行移動生成的曲面都稱為柱面。所以若日後遇到 cylinder 這個字的時候要注意。

在這裡必須了解代數式的操作與幾何圖形的對應關係，若將二次式改用矩陣表達時，利用矩陣對角化的方式，可以把二次式的交叉項 D, E, F 消除，這個操作在幾何圖形的對應是圖形的旋轉；若將方程式進行配方法，則可以把一次項消除，這個操作對應到的幾何概念是圖形的平移。

Six types of quadric surfaces in standard form, page 837



d2R4Kvz-brE

這裡要認識六種非退化的二次曲面，圖形與對應的方程式之間應該要想清楚，透過與坐標平面平行的平面與圖形相截得到的二次曲線可以幫助了解曲面的形狀。這六種二次曲面的認識是爲了之後多變數微積分而準備。多變數微積分的其中一個學習重點是了解曲面長相，像是彎曲的現象與極值的討論。

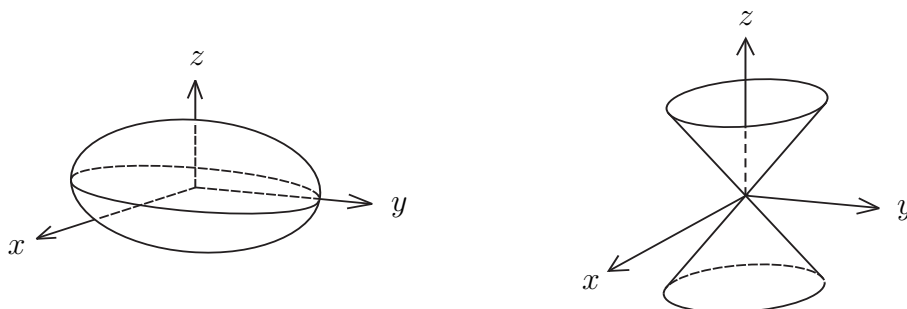


Figure 2: Ellipsoid (橢球) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and cone (錐) $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

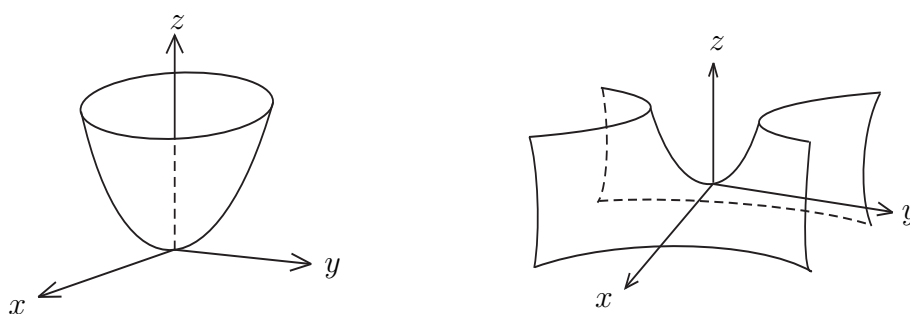


Figure 3: Elliptic paraboloid (橢圓拋物面) $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, $c > 0$ and hyperbolic paraboloid (雙曲拋物面) $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$, $c < 0$.

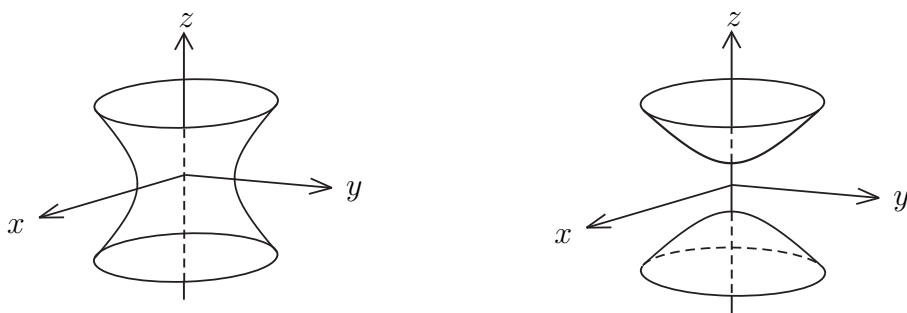


Figure 4: Hyperboloid of one sheet (單葉雙曲面) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ and hyperboloid of two sheets (雙葉雙曲面) $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Exercise (page 840). Classify the following surfaces.

(a) $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$.

(b) $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$.

(c) $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.

(d) $z = x^2 - y^2$.

(e) $y^2 + z^2 = 1 + x^2$.

(f) $-4x^2 + y^2 - 4z^2 = 4$.